New Approach to Alignment Problems

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For any type of tracking detectors there is a necessity in a complex multiparameter transformation of digitized data obtained by a given detector to a standard reference frame. Since each detector is a multicomponent system, the mutual positions of its components may be disturbed on assembly. The resulting misalignment brings about the systematic distortion of recorded data. The mathematical determination and compensation of these distortions (a procedure that is referred to as **alignment**) is indispensable for further correct processing. There are similar procedures for optical track detectors, which are referred to as calibration procedures and using measurements of a special reference, etalon pattern for the transformation of quantities measured by a detector from the inner coordinate system of the detector to the unified laboratory coordinate system. However, for present-day electronic detectors such an approach to using an etalon object for fixing possible distortions is fundamentally impossible because as, first, the alignment problems are of high hierarchy and, secondly, in large electronic detectors, one is forced to use data of ordinary measurements instead of reference data.

Since a detector as a whole can be a vast hierarchical system of numerous complex components, the alignment procedure are also divided into local alignment procedures for individual subdetectors and the global alignment of the entire detector. The approach, which is referred to as **data driven approach** (DDA), where the calculations are controlled by data is also typical for alignment tasks even though it is of necessity used now to calibrate certain components of detectors (e.g., when obtaining the calibration transformation of drift times to drift radii in the system of drift chambers [1]).

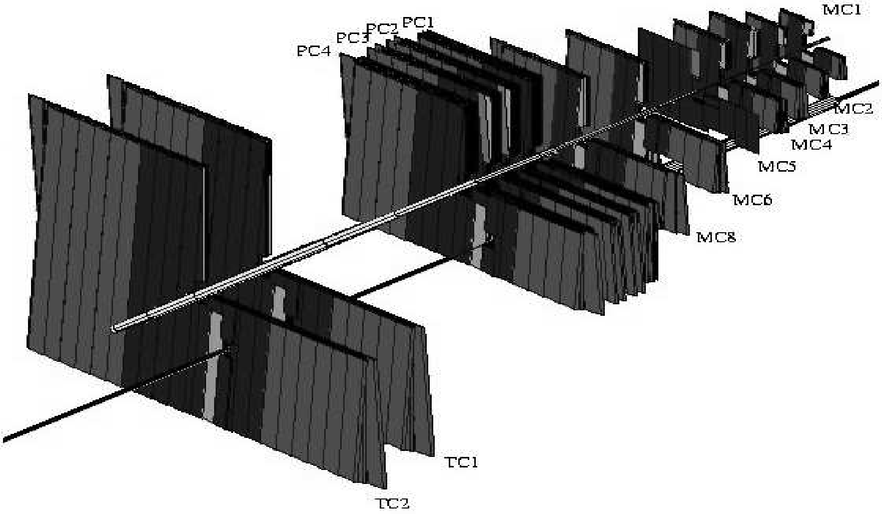

Figure 1: 3D view of the OTR layout

Due to the variety of experiments (collider or with a fixed target) and structures of corresponding detectors, various methods of local and global alignment are applied (see, e.g., [2, 3]). The most recent approach [4] was concerned the Outer TRacker (OTR) of the

HERA-B experiment. The Hera-B Outer Tracker (OTR) Pattern recognition Chamber (PC) is made of 998 separate sensitive planes. These planes are grouped in 24 stereolayers, layers are grouped in 4 superlayers (see fig.1). Some layers are rotated by 80 mrad. Superlayers are divided in 2 parts $(+X)$ and $(-X)$ halves wrapping the proton beam pipe. Each plane has, in principle, 6 degrees of freedom to be displaced, including planar rotation, radial translation and out-of-plane rotation, which require totally 18 parameters to be reconstructed. However their real constructive positions reduce that number considerably. Nevertheless, to determine the remaining parameters for all 998 units, it is necessary to solve an equation system with many thousands of unknown parameters. The situation is even more aggravated due to the necessity to follow the data-driven concept, i.e. to use track information acquired as a result of the fact that the tracks for which alignment is performed emerge from one vertex and simultaneously pass through several chambers. It should be taken into account that tracks in the absence of a magnetic field are straight lines, but their measurements involve errors. In addition, the multiple Coulomb scattering should be taken into account. The **DDA** concept means the use of data sets of real measurements produced by the tracking system. Misalignment of detector modules can be found by analyzing residuals between the measured values and fitted track coordinate. These residuals are functionally dependent on the two types of parameters: track model parameters and alignment parameters. Therefore, the alignment problem can be formulated as a mathematical problem of minimizing a functional summing the squares of all residuals by both types of parameters.

$$\chi^2 = \sum_{events} \sum_{tracks} \sum_{hits} d_i^2/\sigma_i^2 \tag{1}$$

where d_i is hit residual - the difference between measured and fitted track position:

$$d_i = x_{fit} - x_{meas} \tag{2}$$

and σ_{d_i} is accuracy of hit coordinate measurement. Fitted track position depends of estimated track and alignment parameters:

$$x_{meas} = \sum_{i=1}^{n} a_i \cdot d_i + \sum_{j=1}^{\nu} \alpha_j \cdot \delta_j \tag{3}$$

This sum is a function of estimated parameters a, and the solution is obtained from a set of linear equation (so-called normal equations of least squares):

$$Ca = b \tag{4}$$

where C is matrix of derivatives with a dimension equal to the number of estimated parameters. The solution can be found as

$$a = C^{-1}b \tag{5}$$

Since OTR PC geometry is rather complicated and track multiplicity is high, in order to provide accurate results, all the internal parameter correlation should be taken into account, external degrees of freedom should be known and properly fixed. The main difficulties one faces here are the extreme parameter multiplicity and the unconstrained external degrees of freedom. The number of tracks we need to use to get reasonable alignment precision in the order of $10^4 - 10^6$, and the number of alignment parameters for modern detectors in the order of 10^3. Therefore, the matrix of the normal equation is not invertible using common technique. The internal alignment, based on residual functional minimization is blind to effects of geometrical transformations which project tracks into itself. It leads to singularity of normal equation matrix. It is not possible to find a matrix which is inverse for any singular one.

A millepede-like method proposed in [4, 5] makes one fit to all track and alignment parameters simultaneously using one huge error matrix which precisely describes the relationship between track (**local**) parameters δ_i and alignment (**global**) parameters d_i. This method gets around problem's dimension using matrix inversion by partitioning technique. It splits the large inversion into a set of smaller inversions.

The matrix of normal equation C is very sparse and has a regular block structure, with many vanishing sub-matrices. It has three types of contributions from every particular measurements. The first part is a contribution of a symmetric matrix C_l of dimension n (number of global parameters). The second part is a symmetric matrix Γ_i giving the contribution to the big matrix on the diagonal and depending only on the i-th partial track. Third contribution, is a rectangular matrix G_i, contains correlation between global and local parameters.

One can solve the local equation $\Gamma\alpha = \beta$ for each track $\alpha = \Gamma^{-1}\beta$. Using matrix inversion by partitioning huge matrix could be reduced to n normal equations for global parameters

All information about the detector geometry, tracks, all parameters correlation which were encoded in initial huge matrix C now kept in reduced C'. The direct solution $a = C'^{-1}b'$ represents solution vector a with covariance matrix C'^{-1}.

The next big problem caused by the fact that matrix C of normal equation for internal alignment is singular. It has **rank defect**. A very powerful technique, known as **singular value decomposition** was applied in [4, 5] to diagnose singularity of this matrix and eventually to resolve this problem. The solution can be found either by constraining and inverting matrix with Gaussian elimination, or by picking up solution with the smallest norm, provided by a singular value decomposition.

A program making use of the matrix size reduction and singular value decomposition was developed using C++ programming language and included to the HERA-B framework. The examples of alignment problem analysis were elaborated on simulated data.

References

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